Investigating turbulent particulate channel flow with interface-resolved DNS

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Abstract

We have conducted a DNS study of dilute turbulent particulate flow in a vertical plane channel, considering up to 8192 finite-size rigid particles with numerically resolved phase interfaces. The particle diameter corresponds to approximately 9 wall units and their terminal Reynolds number is set to 136. The fluid flow with bulk Reynolds number 2700 is directed upward, which maintains the particles suspended upon average. Two different density ratios were simulated, varying by a factor of 4.5. The corresponding Stokes numbers of the two particles were $O(10)$ in the near-wall region and $O(1)$ in the outer flow. We have observed the formation of large-scale elongated streak-like structures with streamwise dimensions of the order of 8 channel half-widths and cross-stream dimensions of the order of one half-width. At the same time, we have found no evidence of significant formation of particle clusters, which suggests that the large structures are due to an intrinsic instability of the flow, triggered by the presence of the particles. It was found that the mean flow velocity profile tends towards a concave shape, and the turbulence intensity as well as the normal stress anisotropy are strongly increased. The effect of varying the Stokes number while keeping the buoyancy, particle size and volume fraction constant was relatively weak.

Introduction

In the present study we are interested in the effect of solid heavy particles upon the structure of turbulent flow in a vertical plane channel. Past experimental studies of this configuration have revealed that the turbulence intensity can be substantially modified by the addition of particles (Tsuji, Morikawa, and Shiomi 1984; Kulick, Fessler, and Eaton 1994; Suzuki, Ikenoya, and Kasagi 2000). Depending on the exact choice of the various parameters, either enhancement or attenuation can be achieved. However, the underlying mechanisms of the interaction between the two phases have so far not been elucidated in detail, especially in the regime where the smallest length scales of the turbulent flow are comparable to the particle diameter. Moreover, laboratory measurements can often not provide flow data with sufficient detail for the purpose of analyzing the dynamics of the interaction processes.

Previous numerical simulations, on the other hand, have for the most part been limited to the point-particle regime, which loses its validity when the particle Reynolds number becomes appreciable and/or the size of the particles is not negligible compared to the smallest fluid scales (Elghobashi 1994). More specifically, particle wakes cannot be represented in the framework of the point-particle approach.

To our knowledge, the only previous direct numerical simulation (DNS) study of turbulent vertical channel flow with finite-size particles has been performed by Kajishima et al. (2001). However, these authors considered only a small number ($N_p = 36$) of relatively large particles (with a diameter corresponding to 32 wall units) and the angular particle motion was neglected.

In the course of the present study we have performed interface-resolved DNS of turbulent flow in a vertical plane channel configuration involving up to 8192 spherical particles and integrating the equations of motion over $O(100)$ bulk flow time units. In the following section the particular immersed boundary method used in our simulations is reviewed, before we discuss the various validation checks and grid convergence tests which have been performed. We proceed by describing the specific conditions of the simulated flow cases as well as the initialization procedure, before turning to the presentation of the results. We discuss the Lagrangian correlation functions, the dispersion data, the spatial distribution of the dispersed phase, the structure of the carrier phase and finally the Eulerian statistics. The paper closes with a short summary and conclusion.

Numerical Method

The Navier-Stokes equations for an incompressible fluid can be written as:

\[ \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p = \nu \nabla^2 \mathbf{u} + \mathbf{f} \]  
\[ \nabla \cdot \mathbf{u} = 0 \]

where $\mathbf{u}$ is the vector of fluid velocities, $p$ the pressure normalized with the fluid density and $\mathbf{f}$ a volume force term.
The basic idea of the immersed boundary method is to solve these equations in the entire domain $\Omega$, including the space occupied by the solid particles, instead of only considering the interstitial fluid domain $\Omega_f$. For this purpose, the force term $f$ is introduced and formulated in such a way as to impose a rigid body motion upon the fluid at the locations of the solid particles. The main advantage of this approach lies in the possibility to use a fixed computational grid with a simple structure, allowing for efficient numerical solution techniques to be applied.

In the following we will recall the essential points of our specific formulation of the immersed boundary method (Uhlmann 2005a). For this purpose, let us write the momentum equation in semi-discrete form:

$$
\frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} = \text{rhs}^{n+1/2} + \mathbf{f}^{n+1/2},
$$

(2)

where $\text{rhs}$ regroups the convection term, the pressure gradient and the viscous term, the superscripts denoting the time level. The additional force in (2) can be expressed by simply rewriting the equation (Fadlun et al. 2000):

$$
\mathbf{f}^{n+1/2} = \frac{\mathbf{u}^{(d)} - \mathbf{u}^n}{\Delta t} - \text{rhs}^{n+1/2}
$$

(3)

where $\mathbf{u}^{(d)}$ is the desired velocity at any grid point where forcing is to be applied (i.e. at a node inside a solid body). Formula (3) is characteristic for direct forcing methods (Fadlun et al. 2000; Kim et al. 2001), as opposed to formulations which rely on a feed-back mechanism (Lai and Peskin 2000; Goldstein et al. 1993; Höfler and Schwarzer 2000). The drawback of the latter techniques is an often severe restriction of the time step, caused by the time scale of the feedback law itself. Direct forcing methods, on the other hand, are free from this restriction.

However, problems can arise in practice from the fact that the solid-fluid interface seldom coincides with the Eulerian grid lines, meaning that interpolation needs to be performed in order to obtain an adequate representation of the interface. Inspired by Peskin’s original immersed boundary method (Peskin 1972, 2002), we choose to compute the force term at Lagrangian positions attached to the surface of the particles, viz.

$$
\mathbf{F}^{n+1/2} = \frac{\mathbf{U}^{(d)} - \mathbf{U}^n}{\Delta t} - \text{RHS}^{n+1/2},
$$

(4)

where upper-case letters indicate quantities evaluated at Lagrangian coordinates. Obviously, the velocity in the particle domain $S$ is simply given by the solid-body motion,

$$
\mathbf{U}^{(d)}(\mathbf{X}) = \mathbf{u}_c + \omega_c \times (\mathbf{X} - \mathbf{x}_c), \quad \mathbf{X} \in S,
$$

(5)

as a function of the translational and rotational velocities of the particle, $\mathbf{u}_c$, $\omega_c$, and its center coordinates, $\mathbf{x}_c$. The two remaining terms on the right hand side of (4) can be collected as

$$
\mathbf{\tilde{U}} = \mathbf{U}^n + \text{RHS}^{n+1/2}\Delta t
$$

(6)

which corresponds to a preliminary velocity obtained without applying a force term. Its Eulerian counterpart,

$$
\mathbf{\tilde{u}} = \mathbf{u}^n + \text{rhs}^{n+1/2}\Delta t
$$

(7)

is evaluated explicitly.

The final element of our method is the transfer of the velocity (and r.h.s. forces) from Eulerian to Lagrangian positions as well as the inverse transfer of the forcing term to the Eulerian grid positions. For this purpose we define a Cartesian grid $\mathbf{x}_{ijk}$ with uniform mesh width $\Delta x$ in all three directions. Furthermore, we distribute so-called discrete Lagrangian force points $\mathbf{X}_l$ (with $1 \leq l \leq N_L$) evenly on the particle surface. An ‘even’ distribution of points on the surface of a sphere can be obtained in a pre-processing step by an iterative procedure (App. A, Uhlmann 2005a). Using the regularized delta function formalism of Peskin (Peskin 1972, 2002), the Eulerian/Lagrangian transfer can be written as:

$$
\mathbf{\tilde{U}}(\mathbf{X}_l) = \sum_{ijk} \mathbf{\tilde{u}}(\mathbf{x}_{ijk} - \mathbf{X}_l) \Delta x^3, \quad (8a)
$$

$$
f(\mathbf{x}_{ijk}) = \sum_l F(\mathbf{X}_l) \delta_h(\mathbf{x}_{ijk} - \mathbf{X}_l) \Delta V_l, \quad (8b)
$$

where $\Delta V_l$ designates the forcing volume assigned to the $l$th force point. We use a particular function $\delta_h$ which has the properties of continuous differentiability, second order accuracy, support of three grid nodes in each direction and consistency with basic properties of the Dirac delta function (Roma et al. 1999).

It should be underlined that the force points are distributed on the interface between fluid and solid, and not throughout the whole solid domain. The reason for this is efficiency: the particle-related work currently scales as $(D/\Delta x)^2$ instead of $(D/\Delta x)^3$, where $D$ is the particle diameter. The consequences for the efficiency of the forcing due to these two alternative placements of the forcing points have been discussed in a previous study (Uhlmann 2005b).

The above method has been implemented in a staggered finite-difference context, involving central, second-order accurate spatial operators, an implicit treatment of the viscous terms and a three-step Runge-Kutta procedure for the nonlinear part. Continuity in the entire domain $\Omega$ is enforced by means of a projection method.

The particle motion is determined by the Runge-Kutta-discretized Newton equations for translational and rotational rigid-body motion, which are weakly coupled to the fluid equations.

One step of our algorithm can be summed up as follows:

1. compute the explicit velocity estimation $\mathbf{\tilde{u}}$
2. transfer $\mathbf{\tilde{u}}$ to Lagrangian positions at the fluid-solid interfaces
3. compute the force term $\mathbf{F}$
4. transfer $\mathbf{F}$ back to Eulerian grid positions, obtaining $f$
5. solve the Navier-Stokes equations on the fixed grid with the added force term $f$
6. advance the equations for particle motion, using the available force/torque.

The complete set of equations has been given in (Uhlmann 2005a).

During the course of a simulation, particles can approach each other closely. However, very thin liquid inter-particle
The sedimentation of a single spherical particle was computed in an immersed domain which was sufficiently large to allow for sustenance of the turbulent state. In the observation interval of approximately 3.6 bulk flow time units, the instantaneous particle velocities obtained with the different grids match to within 8.3% of the terminal velocity; the second-moment fluid statistics exhibit a maximum relative discrepancy of 7%. From these figures and the above mentioned comparison with experimental data for sedimentation of a single sphere, it was concluded that a resolution of \(D/\Delta x = 12.8\) is sufficient for our present investigation of turbulent particulate channel flow.

In separate simulations it was verified that our DNS code reproduces single-phase turbulence results faithfully. In particular, using a mesh width of \(\Delta x/h = 1/256\) (which corresponds to the value chosen in the following section) gives results in very good agreement with the reference data of Kim, Moin, and Moser (1987). It should be recalled that our grid is uniform and isotropic, making the resolution extremely fine.
in the center of the channel.

| case | $\rho_p/\rho_f$ | $|g|h/u_b^2$ | $St^+$ | $St_b$ | $N_p$ | $\Omega$ |
|------|-----------------|--------------|--------|--------|-------|--------|
| A    | 10              | 1.625        | 67     | 3.75   | 512   | $4h \times 2h \times h$ |
| B    | 2.21            | 12.108       | 15     | 0.83   | 4096  | $8h \times 2h \times h$ |
| B2   | 2.21            | 12.108       | 15     | 0.83   | 8192  | $16h \times 2h \times h$ |

Table 1: Physical parameters in our simulations of particulate flow in a vertical plane channel. In all cases the particle diameter is chosen as $D = h/20$, the global solid volume fraction is set to $\Phi_s = 0.0042$. B2 is a case with identical conditions as case B, except that the streamwise period of the domain was doubled.

<table>
<thead>
<tr>
<th>case</th>
<th>$N_x \times N_y \times N_z$</th>
<th>$\eta p$</th>
<th>$t_{obs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1024 \times 513 \times 256$</td>
<td>32</td>
<td>556</td>
</tr>
<tr>
<td>B</td>
<td>$2048 \times 513 \times 1024$</td>
<td>512</td>
<td>95</td>
</tr>
<tr>
<td>B2</td>
<td>$4096 \times 513 \times 1024$</td>
<td>1024</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 2: Numerical parameters employed in our DNS. $N_i$ is the number of grid nodes in the $i$th direction, $\eta p$ is the number of processors, $t_{obs}$ is the observation interval in bulk flow time units after discarding the initial transient. The grid spacing in all cases is fixed at $\Delta x = h/256$, corresponding to $N_x = 515$ Lagrangian force points per particle, and the time step at $\Delta t = 0.00162 h / U_b$.

Flow Configuration

We are considering particulate flow in a plane channel which is aligned along the vertical direction (cf. figure 1). The fluid is driven in the positive $x$-direction by a mean pressure gradient $\langle p \rangle_x < 0$. The bulk flow Reynolds number is maintained constant at a value of $Re_u = u_b h/\nu = 2700$ ($u_b$ being the bulk velocity), which generates a turbulent flow with $Re_x = u_x h/\nu = 172$ ($u_x$ being the wall shear velocity) in the absence of particles. The particle size is chosen as $D/h = 1/20$, corresponding to $St^+ = 8.6$ in wall units. This regime is of particular interest since the particles are of comparable size to the coherent structures typically found in the buffer layer of wall-bounded turbulent flow and a mutual interaction can therefore be expected. As mentioned in the previous section, we choose a spatial resolution of $D/\Delta x = 12.8$, which corresponds to a mesh width of $\Delta x / h = 1/256$, i.e. $\Delta x^+ = 0.67$ in wall units.

We have further imposed the equality between bulk fluid velocity ($u_b$) and terminal particle velocity ($u_{c,\infty}$). This condition means that the average sedimentation velocity for a large number of particles will be close to zero, supposing that their spatial distribution is near uniform. In practice the average sedimentation velocity takes a slightly positive value as we will see below. From the condition $u_{c,\infty} = u_b$ it follows that the terminal particle Reynolds number (based upon the particle diameter and the relative velocity between phases) measures $Re_{D,\infty} \approx 136$. From the equality between buoyancy and drag forces acting upon an isolated particle, we can form the Archimedes number, which takes the following value:

$$Ar = (D/h)^3 Re_b^2 |g|h/u_b^2 \left( \frac{\rho_p}{\rho_f} - 1 \right) = 13328,$$

where $\rho_p/\rho_f$ is the density ratio between particle and fluid phase and $g$ the gravitational acceleration. This leaves us with one parameter free to choose ($\rho_p/\rho_f$, say), the other one ($|g|h/u_b^2$) being fixed by (9).

We have analyzed two cases with parameter values as given in table 1. The relative particle density in case A is approximately 4.5 times higher than in case B, and the Stokes number (defined as the ratio between particle and fluid time scales) varies accordingly. In particular, we determine the particle time scale from the usual Stokes drag law, $\tau_{SD} = D^2 \rho_p / (18 \eta_f)$, and consider two time scales for the fluid motion: the near-wall scale $\tau^+ = \nu / u^+ \tau^+$ and the bulk flow scale $\tau_b = h / u_b$. Consequently, we can form two Stokes numbers, characterizing the ratio of time scales in the near-wall region, $St^+ = \tau_{SD} / \tau^+$, and in the outer flow $St_b = \tau_{SD} / \tau_b$. Table 1 shows that the present particles have a relatively large response time with respect to the flow in the near-wall region ($St^+ > 10$), but both scales are comparable in the bulk of the flow ($St_b \sim \mathcal{O}(1)$).

The global solid volume fraction has been set to $\Phi_s = 0.0042$, which can be considered at the upper limit of the dilute regime. The two cases were run in computational domains of different sizes, the volume in case B being eight times larger. The number of particles in cases A, B is therefore $N_p = 512, 4096$, respectively. The domain used in case A represents evidently a smaller sample size per instantaneous flow field. However, it allows for simulations over longer time intervals which is of special interest for the computation of slowly evolving statistical quantities acquired along the particle paths. Table 2 shows the global grid size of the simulations, which exceeds $10^9$ Eulerian nodes and $2 \cdot 10^6$ Lagrangian force points in case B, where the work is typically distributed over 512 processors of a PC cluster with fiberoptical interconnect. Finally, we have recently initiated a third case, B2, which is identical to case B, except that the streamwise period of the domain was doubled. We will briefly mention this run in the final discussion.

Initialization of the Simulations

Particles are introduced into fully-developed turbulent single-phase flow at time $t_0$. The initial particle positions form a regular array covering the computational domain. Their initial velocities are matched with the fluid velocities found at $t_0$ at the respective center locations; the initial angular particle velocities are set to zero. During the first time step of the two-phase computation, the coupling algorithm solidifies the fluid occupying the volume of each particle and buoyancy forces set in. Figure 2(a) shows how the average vertical particle velocity $\langle u_z \rangle_p$ (the symbol $\langle \rangle_p$ indicates an
average over all particles) drops from a value close to the bulk fluid velocity to almost zero over an initial interval of approximately 3 (1) bulk time units in case A (B). Let us mention that the long-time averaged mean upward drift of the particles amounts to 0.029ub and 0.058ub, respectively. The data corresponding to the initial transient will not be considered in the following and, therefore, an initial interval of 17 bulk time units has been discarded. The length of the observation interval for the two cases is indicated in table 2.

![Figure 2](image1.jpg)

**Figure 2:** (a) The average vertical particle velocity during start-up of the two-phase simulation. (b) The time evolution of the friction-velocity-based Reynolds number around the start-up time. Particles are added at \( t = t_0 \), case A; --, case B; ---, single-phase flow.

From figure 2(b) it can be seen that the friction velocity increases immediately after the addition of particles, leading to a sharp rise of \( Re_\tau \) which then oscillates around a new mean value of approximately 225 in both cases. The amplitude and frequency of the oscillations is larger in case A due to the smaller box size. This higher friction velocity in the particulate cases reflects the modification of the mean fluid velocity profile as discussed below (cf. figures 13 and 15).

### Lagrangian Correlations

Let us introduce the notation for the different averaging operations employed in the following. \( \langle \xi \rangle_p \) refers to the stream-wise average of any quantity \( \xi \), \( \langle \xi \rangle_{xz} \) is the average over wall-parallel planes and \( \langle \xi \rangle_t \) is the temporal average (over the observation interval given in table 2, except where otherwise stated); the average over time and wall-parallel planes is denoted by \( \langle \xi \rangle = \langle \langle \xi \rangle_{xz} \rangle_t \); as mentioned above, \( \langle \xi \rangle_p \) refers to the average over all particles. The Eulerian space average of quantities related to the dispersed phase are carried out for discrete ‘bins’ in the wall-normal direction, defined as \( I_j = [j - 1, j) \cdot 2h/N_{bin} \forall 1 \leq j \leq N_{bin} \). In practice a value of \( N_{bin} = 40 \) was chosen, such that the width of each bin corresponds to one particle diameter. The time and planewise average of a particle-related quantity \( \xi \) is then computed for each bin by the following formula:

\[
\langle \xi \rangle(y_j) = \frac{\int_0^{t_{obs}} \sum_{i=1}^{N_p} \xi_i(t) \, dt}{\int_0^{t_{obs}} \sum_{i=1}^{N_p} 1 \, dt},
\]

\[
\int_0^{t_{obs}} \sum_{i=1}^{N_p} \xi_i(t) \, dt \int_0^{t_{obs}} \sum_{i=1}^{N_p} 1 \, dt,
\]

\[
(10)
\]
where $y_j$ is the coordinate of the center of the bin $I_j$.

The Lagrangian auto-correlation of particle velocity components is defined as follows:

$$R_{L,p,\alpha}(\tau) = \frac{\langle (u_{c,\alpha}'(t) - u_{c,\alpha}'(t+\tau)) \rangle_{p,t}}{\langle (u_{c,\alpha}'(t))^2 \rangle_{p,t}},$$  

(11)

where $u_{c,\alpha}'(t) = u_{c,\alpha}'(t) - \langle u_{c,\alpha}' \rangle_{t} \delta_{c,1}$ is the instantaneous velocity of the $i$th particle in the $\alpha$ direction, from which the mean streamwise particle velocity at the corresponding bin $y_j$ has been subtracted. As pointed out by Ahmed and Elghobashi (2001), eliminating the mean velocity contribution yields the Lagrangian correlations due to turbulence.

Our present results are shown in figure 3. First we observe that the initial decay of all components of the correlation function is faster in case B than in case A, as can be expected from the lower relative density in the former case. The first zero-crossing of components $\alpha = 1, 2, 3$ occurs approximately at 90, 16, 16 (50, 3, 2) bulk time units in case A (B). In both cases these values exhibit a large difference between the vertical and the two horizontal components, which resembles the strong anisotropy of the near-wall turbulent flow field, as evidenced e.g. by two-point fluid velocity autocorrelations for streamwise separations (cf. Kim et al. 1987, and discussion below). A striking feature of the correlation functions in case B is what seems to be a damped oscillation around the exponentially decaying value, found in all three coordinate directions, with a period of roughly 8 bulk time units. A corresponding small 'bump' can also be found in the curves for case A, approximately for the same time separation as the first local minimum in case B. It can be speculated that this oscillation is a signature of the large streamwise elongated flow structures which have been detected in the present case and which will be discussed below. The fact that this feature is more visible in case B is again a consequence of its lower particle inertia.

The dispersion of particles (mean-square displacement) along the coordinate direction $x_\alpha$, starting from the particle position at time $t_1$ (coinciding with the beginning of the observation interval) is defined as

$$M_{p,\alpha} = \frac{1}{N_p} \sum_{i=1}^{N_p} \left( x_{c,\alpha}(t) - x_{c,\alpha}(t_1) \right)^2, \quad \left( t - t_1 \right) u_b/h,$$

(12)

where the third term in the sum on the right-hand-side corresponds to the mean displacement of particles originating from wall distances pertaining to the same discrete bin as the $i$th particle in the summation (cf. discussion in Ahmed and Elghobashi 2001). Again, subtraction of the equivalent mean displacement yields the dispersion due to turbulence. The quantity $dx_\alpha(y_j)$ is computed analogously to (10), viz.

$$dx_\alpha(y_j) = \sum_{i=1}^{N_p} \frac{x_{c,\alpha}'(t_i) - x_{c,\alpha}'(t_1)}{x_{c,\alpha}'(t_i) \in I_j}.$$

(13)

From figure 4 it can be seen that the dispersion of the particles is largest in the streamwise (vertical) direction and smallest in the spanwise direction. The figure also shows that the dispersion in case B is in all three directions is greater than the counterpart in case A at all times. It is evident that the smaller size of the computational domain starts to affect the time evolution earlier in case A, with the curve for the streamwise component already levelling off after approximately 8 bulk time units. If we concentrate on case B, we observe that the dispersion in the vertical direction follows approximately a behaviour proportional to $t^2$, whereas the two horizontal components grow with slopes intermediate between $t$ and $t^2$.

Structure of the Dispersed Phase

The wall-normal profile of the mean solid volume fraction $\langle \phi_s \rangle$, normalized by the global value $\Phi_s$, is shown in figure 5. The curves are nearly symmetric and very similar in both cases, the data in case B being considerably smoother due to the larger sample size. The distribution of particles is visibly inhomogeneous with a sharp peak located around $y/h = 0.1$, corresponding to approximately 23 wall units. The position of the peak concentration coincides therefore with the location of the maximum intensity of streamwise fluid velocity fluctuations (cf. figure 16 below), commonly associated with the most probable position of buffer-layer streaks. It should be pointed out that we do not observe an increase in the height of the peaks as the simulation proceeds further, indicating that a statistical equilibrium between 'outward' and
‘inward’ wall-normal particle motion has been reached. At the center of the channel, the mean solid volume fraction exhibits a third local maximum at about 70% of the amplitude of the main peaks. However, the variation of \((\phi_s)\) across the bulk flow region (0.5 \(\leq y/h \leq 1\)) is rather small.

In order to gain further insight into the instantaneous spatial distribution of particles, we define the average distance to the nearest neighbor, normalized by the value for a homogeneous distribution. Symbols as in figure 3. The dashed line corresponds to the value for a random distribution with the same solid volume fraction.

Figure 6: Time evolution of the average distance to the nearest neighbor, normalized by the value for a homogeneous distribution. Symbols as in figure 3. The dashed line corresponds to the value for a random distribution with the same solid volume fraction.

\[ d_{min} = \frac{1}{N_p} \sum_{i=1}^{N_p} \min_{j \neq i} (d_{i,j}) , \] (14)

where \(d_{i,j} = |x_i - x_j|\) is the distance between the centers of particles \(i\) and \(j\). The time evolution of the quantity \(d_{min}\), normalized by its value for a homogeneous distribution with the same solid volume fraction, \(d_{hom}^{min} = (||\Omega||/N_p)^{1/3}\), has been plotted in figure 6. The ratio is bounded by the values 0.2 (corresponding to each particle being in contact with at least one other particle) and 1.0. Actual values recorded during our simulations vary mildly between 0.5 and 0.6, which is in fact very close to the value of a random distribution (determined as approximately 0.555). It is interesting to note that simulations of pure sedimentation in triply periodic domains also yield average values of approximately 0.55 for the mean minimum inter-particle distance (Kajishima 2004).

An alternative way of characterizing the spatial structure of the dispersed phase is by searching directly for the presence of particle clusters. Here we define a cluster as a set of particles of which each member is within a distance \(l_c\), at least one other member (Wylie and Koch 2000). The cluster detection has been implemented according to the fast algorithm suggested by Melheim (2005). Figure 7(a) shows our results for case B (data for case A exhibits more noise, but essentially the same features). It can be seen that for two different cut-off lengths \(l_c = 2.5D\) and \(4D\) the probability of finding clusters of a given size does not differ significantly

Figure 7: Probability of finding a particle cluster with \(n_c\) members in case B (symbols). (a) Using two different spherical cut-off lengths of \(l_c = 2.5D\) and \(4D\); (b) using an ellipsoidal cut-off with axes of length \(l_{cx} = 7.5D\), \(l_{cy} = l_{cz} = 2.5D\). The probability was evaluated from 20 instantaneous snapshots. The dashed lines correspond to the probability for a random particle distribution with the same solid volume fraction (evaluated from 100 realizations).
from the corresponding probability found in random particle distributions. Since particles can interact over long distances via their wakes, the cluster definition was then modified in order to take the non-isotropic structure of the wakes into account. Instead of using a spherical cut-off radius, we define clusters through an ellipsoidal definition. More specifically, two particles \(i\) and \(j\) are considered as members of a cluster if their positions obey

\[
\left(\frac{x_{c,1}^{(i)} - x_{c,1}^{(j)}}{l_{cz}}\right)^2 + \left(\frac{x_{c,2}^{(i)} - x_{c,2}^{(j)}}{l_{cy}}\right)^2 + \left(\frac{x_{c,3}^{(i)} - x_{c,3}^{(j)}}{l_{cz}}\right)^2 \leq 1.
\]

Figure 7(b) shows the probability of finding clusters with an elongation by a factor of 3 in the streamwise direction. The probability increases with respect to the spherical definition, but so does the result for the random particle distribution. It must therefore be concluded that the spatial distribution of particles in the present cases is not significantly different from a random distribution.

**Structure of the Carrier Phase**

Figure 8 shows isosurfaces of the fluctuations (with respect to the plane-average) of the streamwise velocity field for one snapshot of case B and for single-phase flow. The flow in the absence of particles exhibits the well-known streaky structures with streamwise characteristic lengths between 200 and 600 wall units. In the two-phase case the picture is radically different. First, a large number of particle wakes clutter the image, being recognizable as streamwise elongated surfaces with negative fluctuation values (due to fluid upflow and negative particle buoyancy). In addition, we can observe very large coherent structures of both signs, with streamwise dimensions of the order of the length of the current domain \((8h)\). These structures are clearly induced by the addition of particles since they have no counterpart in single-phase flow.

Figure 9 shows a short time-sequence of plots of the streamwise-averaged \(x\)-component of the flow velocity from which the plane-average has been subtracted, \(u'' = \langle u \rangle_x - \langle u \rangle_{xz}\), case B. The color code ranges from \(u'' = -3.5u_\tau\) (blue) to \(u'' = +3.5u_\tau\) (red). Time increases from top to bottom, with an increment of approximately 7 bulk flow time units.
Figure 10: As figure 9, but for single phase flow.

$R_\alpha(r_x)$ and $R_\alpha(r_z)$, in wall-parallel planes located approximately 21 wall units above the wall. These quantities were computed from 9 instantaneous flow fields and using planes adjacent to both walls. It should also be noted that the correlations were computed by Fourier transform of the composite flow field, i.e. containing the regions of the immersed solid particles (cf. discussion in the following section). For small streamwise separations $r_x^+$ (figure 11a) all velocity components exhibit a faster decrease than the single-phase case. For streamwise separations over 200 wall units, however, the streamwise component in case B shows a larger correlation value, consistent with the observed large-scale elongated structures. Considering spanwise separations $r_z^+$ (figure 11b) it can be seen that the correlations in the particulate case still exhibit some of the features which are commonly believed to be the signatures of the buffer-layer coherent structures, i.e. a local minimum for separations of approximately 30 wall units for the wall-normal component. However, the correlation functions of the streamwise and spanwise components in the two-phase case do not take the usual negative values for $r_z^+ \approx 50$. Instead, the first zero-crossing takes place at around 150 wall units, and noticeable negative correlation values are found for $R_1$ at separations as large as $r_z^+ \approx 300$. The evidence from the correlation functions shows that the structures which we have observed instantaneously have a large significance for the flow statistics, masking in part even the signatures of the streaks and streamwise vortices.

In order to conclude this section, we consider the effect of the large scale fluid structures upon the particle motion. A visualization of instantaneous particle positions and their velocities is presented in figure 12 for three snapshots corresponding to the sequence of flow fields of figure 9. Here particles with large positive (negative) streamwise velocity fluctuation with respect to the long-time average are highlighted by red (blue) color. Comparing figures 12 and 9, a strong spatial correlation between locally high (low) fluid velocity and high (low) particle velocity can be observed. Continuous animations of the three-dimensional particle motion of this case show that the particles indeed exhibit collective motion in form of streamwise-elongated patches over times of the order of 20 bulk units.

Figure 11: Two-point autocorrelations of flow velocity fluctuations at a wall distance of $y/h = 0.094 \ (y^+ \approx 21)$ in case B. (a) For streamwise separations; (b) for spanwise separations. Colors indicate the components ($\alpha = 1, 2, 3$). The correlations were obtained by averaging over 18 instantaneous data sets spanning a temporal interval of 45 bulk time units. The dashed lines correspond to single-phase flow of Kim et al. (1987).
Figure 12: Instantaneous particle positions, projected upon the cross-flow plane, corresponding to the flow fields in figure 9. The particles are colored according to their instantaneous vertical velocity fluctuations (with respect to the long-time average): \( u_c - \langle u_c \rangle \geq 0.18 u_b \), red; \( u_c - \langle u_c \rangle \leq -0.18 u_b \), blue; else, grey.

Eulerian Statistics

Here we consider the Eulerian statistics which have been accumulated over time and wall-parallel planes, as indicated above. Concerning the computation of statistical quantities of the carrier phase, no distinction has been made between the fluid regions and the volume occupied by the solid particles, as already mentioned during the discussion of figure 11. This decision was taken in favor of computational efficiency. The difference between the present approach on the one hand and accumulation of statistics exclusively at grid nodes occupied by the fluid phase on the other hand has been evaluated from 5 instantaneous snapshots in case B. The differences were found to be insignificant, except for the r.m.s. of streamwise velocity fluctuations, where a maximum relative discrepancy of 6.3% was found. In fact, the current approximate evaluation of the statistics always overestimates the r.m.s. value of the streamwise fluid velocity fluctuations and slightly underestimates the mean streamwise fluid velocity. This is due to the fact that the average rigid body motion represents a negative velocity fluctuation with respect to the mean of the carrier phase at all wall distances.

The usual check of the statistical convergence of low-order moments in plane channel flow includes a verification of linearity of the total shear stress \( \tau_{tot} = \langle u'v' \rangle^+ - \partial_y \langle u \rangle^+ \). However, integrating the streamwise momentum balance over the whole domain and time gives for the present case (assuming a statistically steady state):

\[
\frac{1}{Re_x} \partial_{yy} \langle u \rangle^+ - \partial_y \langle u'v' \rangle^+ + 1 = \left( \frac{\rho_p}{\rho_f} - 1 \right) \Phi_s - \langle \phi_s \rangle \frac{Fr_{\tau}}{Fr_{\tau}}.
\]

(16)

Here \( \Phi_s - \langle \phi_s \rangle \) represents the local deviation from a homogeneous particle distribution, and we have defined a Froude number based upon the wall shear velocity, \( Fr_{\tau} = u_c^2/(h|g|) \). It can be seen from (16) that whenever the particle distribution is not uniform, the total shear stress does not vary linearly with \( y \). Indeed the present profiles of \( \tau_{tot} \) (not shown) are quite different from the usual straight line, especially due to the distinct peaks in solid volume fraction (cf. figure 5). Therefore, an alternative criterion for the convergence of statistics needs to be considered.

Asymptotically, the profiles of the various first and sec-
The dashed line corresponds to the single-phase reference data of Kim et al. (1987).

Figure 15: The mean flow velocity of case B in wall units. The fact that the low-order moments converge at such a slow rate can be attributed to several causes. First, the present suspension is relatively dilute, which means that it takes a very long time for a sufficient number of particles to visit each wall-normal bin. Secondly, the large-scale flow structures which were discovered in the preceding section impose a very long time scale. The relatively high mean particle velocities around $y/h = 0.4$ (figure 14) are a direct consequence of the slowly evolving high-speed flow structures observed in figures 9 and 12 at the corresponding locations. At the present time it does not seem feasible to continue the integration of these simulations for intervals which would allow the statistics to reach a truly asymptotic form. However, the statistical data accumulated up to the present date is still of considerable interest.

Turning again to the mean velocity profiles for the two phases (figures 13 and 14) we note that there is a strong similarity in shape. As a consequence, the mean slip velocity $\langle u \rangle - \langle u_e \rangle$ is nearly constant across most of the channel width. Its value is approximately $u_e$, as estimated a priori from the equilibrium between buoyancy and drag acting on a single sphere in unperturbed flow. Concerning the mean fluid velocity in figure 13, we observe a clear difference with respect to the single-phase case, characterized by a tendency to form a concave profile with larger gradients at the walls and a flat section near the centerline. Concave velocity profiles were previously observed by Tsuji et al. (1984) in experiments on vertical particulate pipe flow, albeit at higher Reynolds numbers and much higher density ratios. The presently simulated cases are therefore situations where a strong two-way interaction takes place.

Figure 16: The normal stresses of case B. Colors indicate the velocity components: $\alpha = 1$, $\alpha = 2$, $\alpha = 3$. The dashed lines correspond to the single-phase reference data of Kim et al. (1987).

Figure 15 shows the mean velocity profile of case B in wall scaling and semi-logarithmic representation. It can be seen that the logarithmic region is shorter than in single-phase flow and that the intercept of the logarithmic law is substantially decreased, similar to rough-wall turbulent flow. This behaviour is consistent with the increase in the friction-velocity based Reynolds number observed in figure 2(b).

The r.m.s. fluctuations of fluid velocities are shown in figure 16. The largest difference with respect to the single-phase data is obtained for the streamwise component which is practically constant in the range $0.25 \leq y/h \leq 1$. When normalized by wall units, the wall-normal and spanwise components are reduced in the particulate case across most of the channel width. The overall turbulence intensity is increased substantially, i.e. by a factor of approximately 6 at the centerline, and the anisotropy is strongly affected. The injection of energy into the streamwise fluctuations has two causes: the presence of wakes, and the formation of large-scale streamwise elongated flow structures. Both phenomena were visualized in figure 8. Experimental measurements of Suzuki et al. (2000) in more dilute downward vertical channel flow at comparable Reynolds numbers and similar values for the particle parameters have also revealed a substantial increase in streamwise fluid velocity fluctuation levels and anisotropy for $y^+ > 20$.

The Reynolds stress profile (cf. figure 17) exhibits an extensive region with values close to zero ($0.6 \leq y/h \leq 1$). In this outer region the presence of particles apparently causes a decorrelation between the turbulent motion in the streamwise and wall-normal direction, thereby suppressing the average Reynolds stress.

Finally, let us turn to some Eulerian statistics pertaining to the dispersed phase. Figure 18 shows the r.m.s. particle velocity fluctuations of case B. A similar trend as for the fluid velocities can be noted with a near-equality of the two horizontal components and a much larger intensity of the vertical component. Furthermore, all components exhibit a small peak at approximately $y^+ = 34$ and a visibly lower intensity around the common location of the buffer-
layer streaks. Compared to the fluid counterpart, the intensity of the streamwise particle velocity fluctuations is approximately 25% lower across most of the channel, while it is slightly higher for the two horizontal components. Regarding this comparison between the two phases, Suzuki et al. (2000) have observed smaller intensities of the particle velocity fluctuation for all components above \( y^+ = 20 \) and the opposite behaviour for the streamwise and wall-normal components below this wall-distance.

The profile of the mean angular particle velocity in the spanwise direction \( \langle \omega_{c,z} \rangle \) is shown in figure 19. Its absolute value has a maximum at a wall-distance of approximately 30 wall units and tends towards zero at the centerline and at the wall. The figure also includes a curve given by \(-A \partial_y \langle u \rangle\), which corresponds to the steady rotation of a cylinder in a simple shear flow, where \( A \) is positive and decreases with the shear Reynolds number (cf. Ding and Aidun 2000; Zettner and Yoda 2001). It can be seen that our data for case B is reasonably well represented when choosing \( A = 0.1 \), except for the region \( y^+ < 30 \), where the damping effect of the wall presumably plays an important role. A similar conclusion is reached for the data of case A (not shown). Constrained simulations of single neutrally-buoyant particles in laminar pipe flow have likewise yielded a proportionality between the steady-state angular particle velocity and the wall-normal fluid velocity gradient over most of the pipe radius (Yang et al. 2005, figure 3).

**Summary and Discussion**

We have conducted a DNS study of turbulent particulate channel flow in a vertical arrangement, considering finite-size rigid particles with numerically resolved phase interfaces. The immersed boundary simulations were performed for the dilute regime (0.42% solid volume fraction), allowing for an approximate treatment of direct particle encounters. Our simulations are for spherical heavy particles whose buoyancy is adjusted such that the terminal velocity matches the bulk flow velocity. For the present bulk Reynolds number of 2700 and particle size of 1/20 of the channel half-width (corresponding to approximately 9 wall units), this leads to a terminal particle Reynolds number of 136. Two different density ratios were considered, varying by a factor of 4.5. The corresponding Stokes numbers of the two particles were \( O(10) \) in the near-wall region and \( O(1) \) in the outer flow.

We have presented Lagrangian autocorrelations of particle velocities, particle dispersion, particle distribution functions, flow visualizations, Eulerian two-point autocorrelation data and Eulerian one-point moments. The main findings can be summarized as follows:

- We observe the formation of large-scale elongated streak-like structures with streamwise dimensions of the order of 8 channel half-widths and cross-stream dimensions of the order of one half-width. Particle velocities are strongly affected by these structures, which in turn allows to follow their evolution over substantial time intervals. Moreover, the large particle-induced structures alter the fluid velocity auto-correlations substantially.

- We have found no evidence of significant cluster formation of the dispersed phase. This conclusion is based upon simple visualization, monitoring of the mean min-
The presence of slowly-evolving large scales leads to a slow convergence of the Eulerian statistics, which is evidenced by the lack of symmetry of the profile of the mean fluid velocity, amongst others, even after a considerable observation interval (556 and 95 bulk time units in cases A and B, respectively).

The presence of particles causes a strong modification of the mean flow, which tends towards a concave profile, as well as a substantial increase in turbulence intensity (by up to a factor of 6), mainly in terms of streamwise velocity fluctuations.

The effect of varying the Stokes number while keeping the buoyancy, particle size and volume fraction constant was relatively weak. Only the Lagrangian correlations and the dispersion were significantly affected.

Vertically elongated structures have previously been detected in pure sedimentation flows by Kajishima and Takiguchi (2002) (see also Kajishima 2004) and in grid-generated turbulence by Nishino and Matsushita (2004). However, in the aforementioned cases the main feature of the “structures” was a local accumulation of particles. This is not true in the present case, where we have been unable to detect the large structures by considering only the spatial distribution of particles. Instead we believe that the addition of particles to the turbulent wall-bounded flow triggers an inherent instability leading to the formation of streak-like structures of very large dimensions. This specific point should be further investigated in the future.

One question which naturally arises in relation with the emergence of flow scales of the order of the streamwise period of the computational domain is whether the finite box size might play a decisive role. It should be mentioned that these largest scales are not clearly observed in the case with the smaller domain (case A). On the other hand, performing simulations with much larger domains than the one used in case B is a challenging task, since the simulations in case B are already on the limit of what we can afford with the present algorithm on the largest available hardware. Nevertheless, we have recently initiated a companion simulation B2 (cf. tables 1 and 2) with twice the streamwise period as case B and otherwise identical conditions. The run was initialized with an exact periodic extension of a fully developed field of case B (towards the end of the observation interval). Up to the present date (after an elapsed time of 11 bulk time units), the results of this extended simulation have not significantly diverged from the original case B. It still remains to be seen how the large streak-like structures scale in the extended domain, once the simulation has sufficiently evolved from the initial condition.

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