Modelling of turbulent flows: RANS and LES
Turbulenzmodelle in der Strömungsmechanik: RANS und LES

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Lecture 1
LECTURE 1:
Introduction to turbulence modelling
Questions to be answered in the present lecture

What are the salient features of turbulence?

Which principle approaches exist for its computation?
Why does the engineer care about turbulent flow?

Design requires description/prediction of

- hydrodynamic forces
  - vehicles, buildings, pipes...
- heat transfer
  - combustion chambers, ocean surface, nuclear plants...
- mixing
  - combustion processes, waste water treatment, chemical engineering...
- acoustics
  - noise generation by vehicles, engines, hydraulics...

⇒ strongly influenced by turbulence!
The challenge of turbulence

Recap of the salient features of turbulent flows

- 3D, time-dependent, random flow field
- largest scales are comparable to characteristic flow size → geometry-dependent, not universal
- wide range of scales: $\tau_\eta/T \sim Re^{-1/2}$, $\eta/L \sim Re^{-3/4}$
- wall flows: energetic motions scale with viscous units $\delta_\nu/h \sim Re^{-0.88}$
- non-linear & non-local dynamics
Effects of turbulence can be desired or undesired

<table>
<thead>
<tr>
<th>Effects of increased properties</th>
<th>not desired</th>
<th>desired</th>
</tr>
</thead>
<tbody>
<tr>
<td>momentum:</td>
<td>drag increase</td>
<td>delayed separation</td>
</tr>
<tr>
<td>mass:</td>
<td>controllability</td>
<td>mixing in combustion,</td>
</tr>
<tr>
<td>of processes</td>
<td></td>
<td>dispersion of pollutants</td>
</tr>
<tr>
<td>heat:</td>
<td>heat loss</td>
<td>efficient exchange</td>
</tr>
</tbody>
</table>

⇒ Turbulent flow control is an important field

- aeronautics, turbomachinery, . . .

Also: stability, transition
General criteria for assessing turbulence models

Level of description

▶ how much information can be extracted from the results?

Computational requirements & development time

▶ how much effort needs to be invested in the solution?

Accuracy

▶ how precise and trustworthy are the results?

Range of applicability

▶ how general is the model?
Possible discrepancies between computation & experiment

(adapted from Pope, 2000)
Different approaches to computing turbulent flows (1)

Time-dependent, 3D solution of Navier-Stokes:

“Direct numerical simulation” (DNS)

- given the field \( u(t=0) \) and boundary conditions
- integrate in time, then compute desired quantity

Problems:

- sensitivity to initial conditions → statistics
- enormous computational requirements
Different approaches to computing turbulent flows (2)

Spatially filtered Navier-Stokes:

“Large eddy simulation” (LES)

- evolution of large scales is simulated
- small-scales are modelled
→ substantial savings w.r.t. DNS (coarser grid, larger step)

Problems:

- 3D, time-dependent
- still costly in practice
Different approaches to computing turbulent flows (3)

Reynolds-averaged Navier-Stokes (RANS)

- Equations are averaged \textit{a priori}
- Turbulence appears as additional unknowns
- Often stationary problem, additional symmetries

Problems:

- Closure problem
- Modelling required
Different approaches – Spectral view

- recall Komogorov hypothesis and scaling
General methods for computing averages

- probabilistic definition
  \[ \langle u(t) \rangle \equiv \int_{-\infty}^{\infty} V f(V, t) \, dV \]

- ensemble averaging (repeated experiments):
  \[ \langle u(t) \rangle_N \equiv \frac{1}{N} \sum_{n=1}^{N} u^{(n)}(t) \]

- time averaging (statist. stationary flow):
  \[ \langle u(t) \rangle_T \equiv \frac{1}{T} \int_{t}^{t+T} u(t') \, dt' \]

- space averaging (homogeneous flow):
  \[ \langle u(t) \rangle_{L} \equiv \frac{1}{L^3} \int_{L} u(x, t) \, dx \]

...
Spectral view: DNS, LES, RANS

- DNS resolves dissipative scales, LES resolves energetic scales
- RANS resolves mean flow (i.e. the geometry)
Different approaches – Hierarchy

- direct num. simulation (DNS)
- large-eddy simulation (LES)
- RANS

accuracy, simulation cost

modelling complexity
From fluid mechanics to turbulent flow computation

Stages of the analysis:

<table>
<thead>
<tr>
<th>fluid mechanical process</th>
<th>mathematical model</th>
<th>discrete equations</th>
<th>computational code</th>
</tr>
</thead>
<tbody>
<tr>
<td>physics</td>
<td>numerical</td>
<td>mathematics</td>
<td>programming</td>
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Interdependence of the three stages
Introduction to turbulence modelling

Characteristics of turbulence

Computing turbulent flows – approaches

History

History of turbulence modelling/simulation

1877 Boussinesq: concept of turbulent eddy viscosity
1925 Prandtl: mixing length hypothesis
1945-51 Chou, Rotta: Reynolds-stress transport model
1963 Smagorinsky: LES with sub-grid model
1968 Harlow & Nakayama: $k-\varepsilon$ model
1970 Deardorff: LES of plane channel flow
1972 Orszag & Patterson: DNS of hom.-iso. turbulence
1987 Kim, Moin & Moser: DNS of plane channel flow
Summary

Introduction to turbulence modelling

- DNS resolves all scales up to the dissipative range
- LES resolves the energy-containing scales
- RANS resolves only the mean flow
Outlook on next lecture: DNS as numerical experiments

What are the purposes of performing DNS?

What are its capabilities and limitations
Further reading